INTEGRATION

1 Evaluate

a $\int_1^3 (4x - 1)\ dx$ **b** $\int_0^1 (3x^2 + 2)\ dx$ **c** $\int_0^3 (x - x^2)\ dx$

d $\int_2^3 (3x + 1)^2\ dx$ **e** $\int_1^2 (x^2 - 8x - 3)\ dx$ **f** $\int_{-2}^4 (8 - 4x + 3x^2)\ dx$

g $\int_1^4 (x^3 - 2x - 7)\ dx$ **h** $\int_{-2}^{-1} (5 + x^2 - 4x^3)\ dx$ **i** $\int_{-1}^2 (x^4 + 6x^2 - x)\ dx$

2 Given that $\int_1^4 (3x^2 + ax - 5)\ dx = 18$, find the value of the constant a.

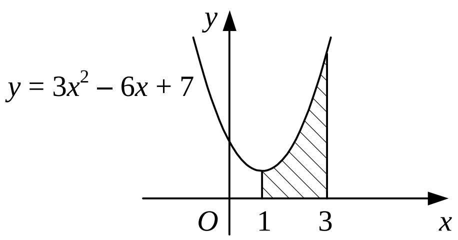
3 Given that $\int_{-1}^k (3x^2 - 12x + 9)\ dx = 16$, find the value of the non-zero constant k.

4 Evaluate

a $\int_1^3 (2 - \frac{1}{x^2})\ dx$ **b** $\int_{-2}^{-1} (6x + \frac{4}{x^3})\ dx$ **c** $\int_1^4 (3x^{\frac{1}{2}} - 4)\ dx$

d $\int_{-1}^2 \frac{4x^4 - x}{2x}\ dx$ **e** $\int_1^8 (x - x^{-\frac{1}{3}})\ dx$ **f** $\int_2^3 \frac{1 - 6x^3}{3x^2}\ dx$

5



The diagram shows the curve with the equation $y = 3x^2 - 6x + 7$.

Find the area of the shaded region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 3$.

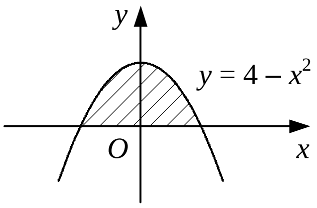
6 Find the area of the region enclosed by the curve $y = f(x)$, the x-axis and the given ordinates.
In each case, $f(x) > 0$ over the interval being considered.

a $f(x) \equiv x^2 + 2$, $x = 0$, $x = 2$ **b** $f(x) \equiv 3x^2 + 8x + 6$, $x = -2$, $x = 1$

c $f(x) \equiv 9 + 2x - x^2$, $x = 2$, $x = 4$ **d** $f(x) \equiv x^3 - 4x + 1$, $x = -1$, $x = 0$

e $f(x) \equiv 2x + 3x^{\frac{1}{2}}$, $x = 1$, $x = 4$ **f** $f(x) \equiv 3 + \frac{5}{x^2}$, $x = -5$, $x = -1$

7



The diagram shows the curve with the equation $y = 4 - x^2$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Find the area of the shaded region enclosed by the curve and the x-axis.

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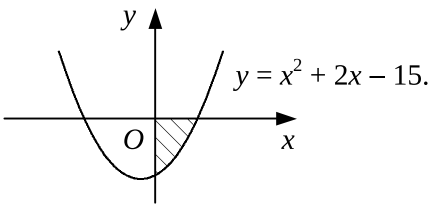
8 In each part of this question, sketch the given curve and find the area of the region enclosed by the curve and the x-axis.

a $y = 6x - 3x^2$ **b** $y = -x^2 + 4x - 3$ **c** $y = 4 - 3x - x^2$ **d** $y = 2x^{\frac{1}{2}} - x$

9 **a** Sketch the curve with the equation $y = x^2 + 4x$.

b Find the area of the region enclosed by the curve, the x-axis and the line $x = 2$.

10



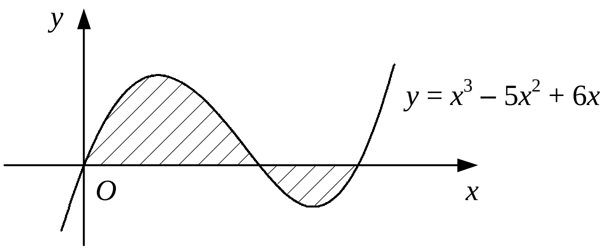
The diagram shows the curve with the equation $y = x^2 + 2x - 15$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Evaluate the integral $\int_0^3 (x^2 + 2x - 15)\ dx$.

c State the area of the shaded region enclosed by the curve, the y-axis and the positive x-axis.

11

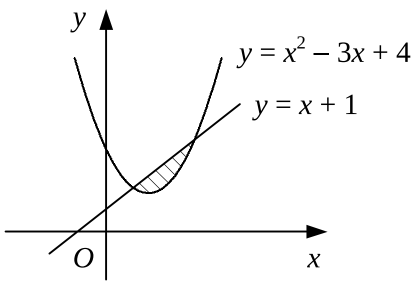


The diagram shows the curve with the equation $y = x^3 - 5x^2 + 6x$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Show that the total area of the shaded regions enclosed by the curve and the x-axis is $3\frac{1}{12}$.

12



The diagram shows the curve $y = x^2 - 3x + 4$ and the straight line $y = x + 1$.

a Find the coordinates of the points where the curve and line intersect.

b Find the area of the shaded region enclosed by the curve and the line.

13 In each part of this question sketch the given curve and line on the same set of coordinate axes and find the area of the region enclosed by the curve and line.

a $y = 9 - x^2$ and $y = 6 - 2x$ **b** $y = x^2 - 4x + 4$ and $y = 16$

c $y = x^2 - 5x - 6$ and $y = x - 11$ **d** $y = \sqrt{x}$ and $x - 2y = 0$